THE LENS LAW

AN AUTOFOCUS MECHANISM

INTRODUCTION

An autofocus mechanism is a device which allows to change the magnification of an optical system by moving the object, image plane and even the lens, still maintaining the image always at focus.

This task can be accomplished by using a simple mechanical device, a sort of an analog computer.

Three techniques are reported here: *the three hinged rods*, *the endless belt*, and *the sliding rods* mechanisms.

These mechanical autofocus mechanisms are used in some photographic enlarger and photocopier.

The Lens law

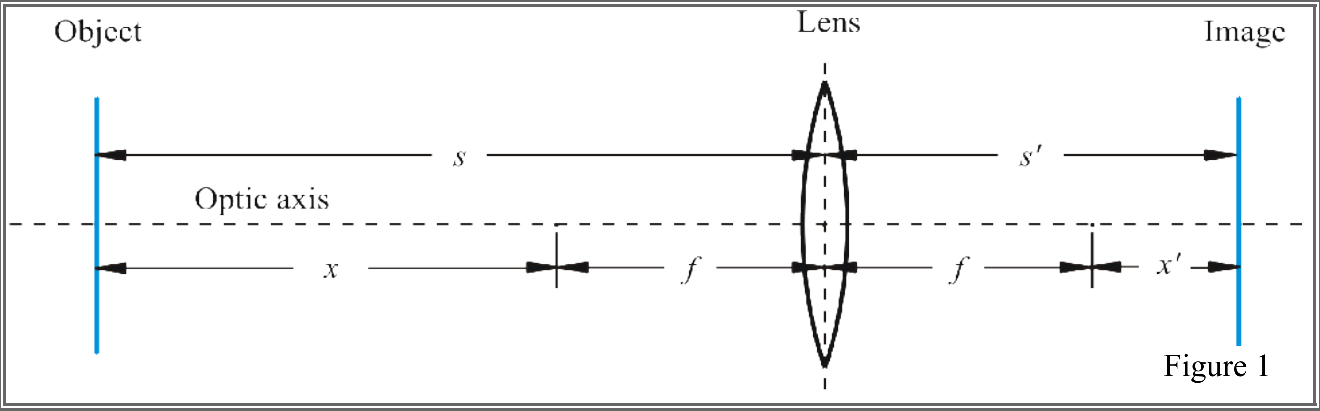


Figure 1

$$\boxed{\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}}$$

f, focal length. Substituting: $x=s-f$ and $x'=s'-f$, we obtain the equivalent Newton formula:

$$\boxed{x\cdot x'=f^2}$$

The system magnification is:

$$\boxed{M=-\frac{s'}{s}=-\frac{x'}{f}=-\frac{f}{x}}$$

The three hinged rods mechanism

Three rods are hinged together at their vertexes (C), the other extremities are linked to the object, lens and image plane respectively, as shown in Fig. 2.

The object and image plane are located a distance f (the lens focal length) apart from the respective hinged joints with their rods.

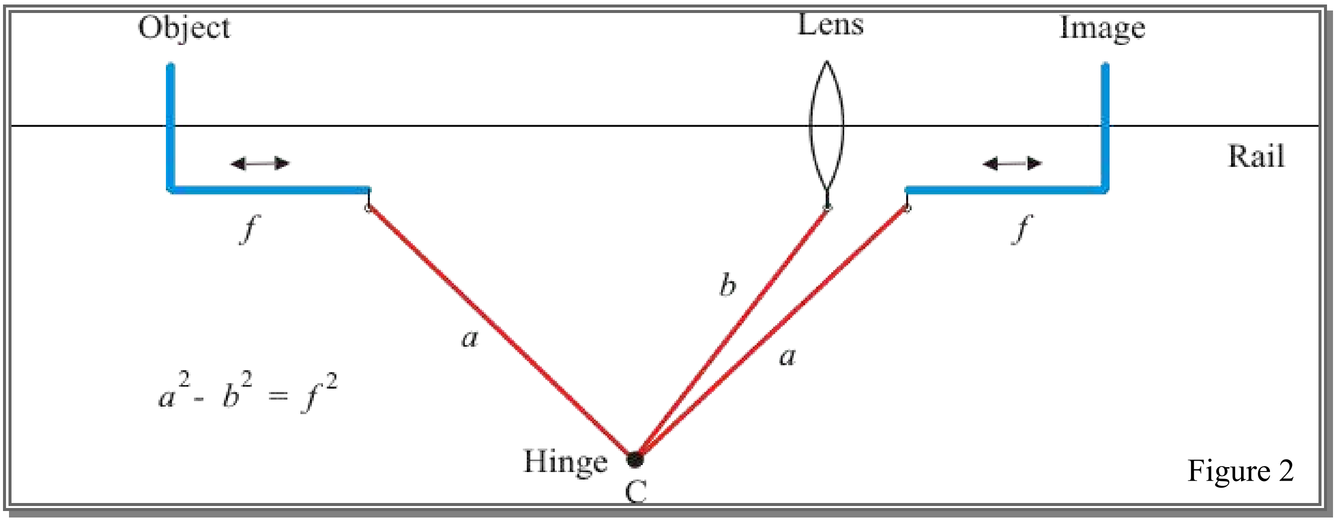


Figure 2

The rods which are linked to the object and image plane have the same length a while the lens rod has a length b. The lens law will be fulfilled if a and b obey to the following equation (see the inset at the end of this paragraph):

$$\boxed{a^2 - b^2 = f^2}$$

The object, the lens and the image plane can move along the optic axis sliding on a rail and the distances among them always fulfill the lens law due to the bond with the three rods. This means that the image will be always at focus.

The magnification M can be tuned from a minimum value of (Fig. 3a):

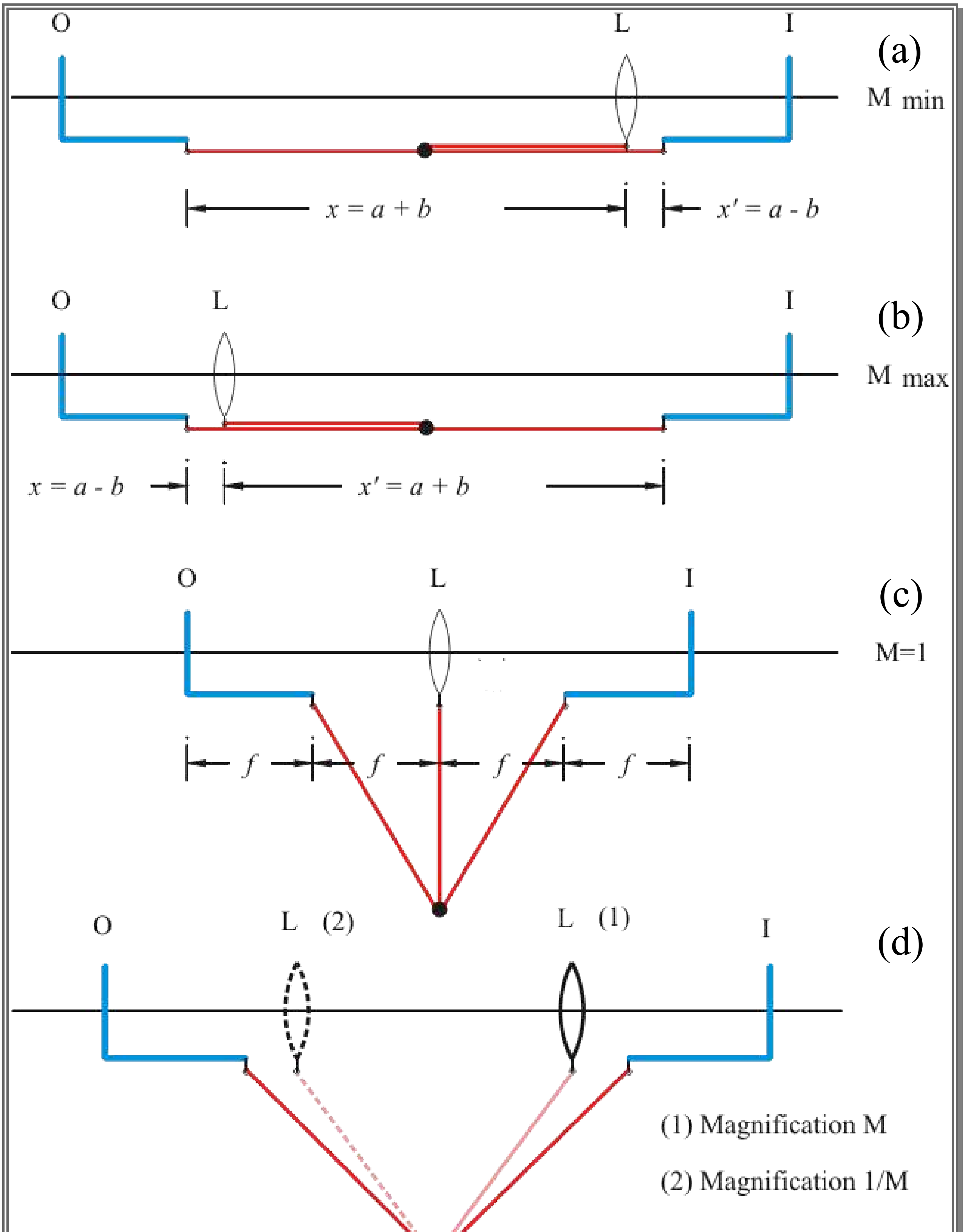
$$\boxed{M_{\min} = -\frac{a-b}{f} = -\frac{f}{a+b}}$$

and a maximum one of (Fig. 3b):

$$\boxed{M_{\max} = \frac{1}{M_{\min}}}$$

Fig. 3c shows the symmetric configuration with magnification M=1.

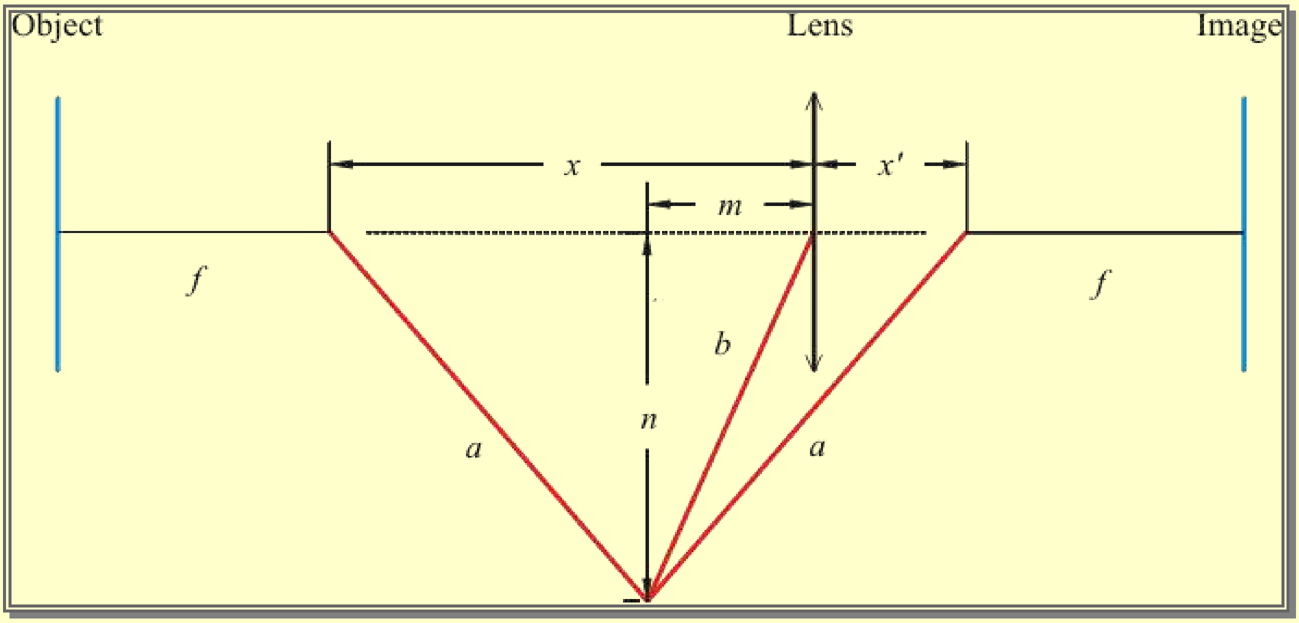
For every configuration where the lens is nearer to the object than to the image plane M > 1. Otherwise M < 1. For each configuration there exists a symmetrical one as shown in Fig. 3d.



O
L
I
(a)
M min
x = a + b
x′ = a - b
O
L
I
(b)
M max
x = a - b
x′ = a + b
O
L
I
(c)
M=1
f
f
f
f
O
L (2)
L (1)
I
(d)
(1) Magnification M
(2) Magnification 1/M

Figure 3

Demonstration of the equivalence between the *rods relation* and the *lens law*



1) $\boxed{a^2 - b^2 = f^2}$ the *rods relation*

Moreover, from the configuration geometry (see the figure:

2) $\boxed{n^2 = a^2 - (x - m)^2}$

3) $\boxed{n^2 = b^2 - m^2}$

4) $\boxed{n^2 = a^2 - (x' + m)^2}$

Subtracting eq. 4) from eq. 2):

5) $\boxed{m = \dfrac{x - x'}{2}}$

Subtracting eq. 3) from eq. 2) and taking into account eq. 1:

6) $\boxed{f^2 = x \cdot (x - 2\mathrm{m})}$

Substituting eq. 5) we finally obtain the Lens law:

$$\boxed{x \cdot x' = f^2}$$

Other autofocus mechanisms

The belt mechanism

The device is made up of an endless belt which turns around two pulleys. The object and image planes are linked to the belt at two points as shown in Fig. 4.

The lens is connected to another belt which is in turn connected to the principal one. This auxiliary belt turns around a third pulley which is located at a distance f from the optical axis.

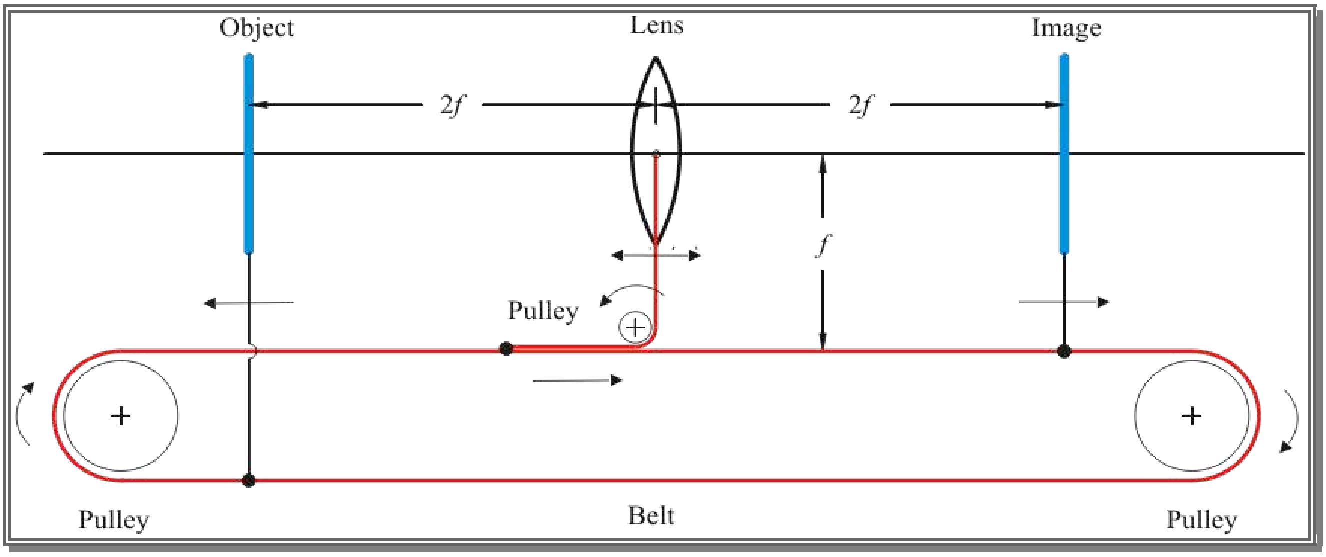


Figure 4

At first the device is set up in the configuration shown in Fig. 4 (with a magnification M=1). When the belt turns (initially in a clockwise direction), the object and the image planes move in opposite directions (Fig. 5).

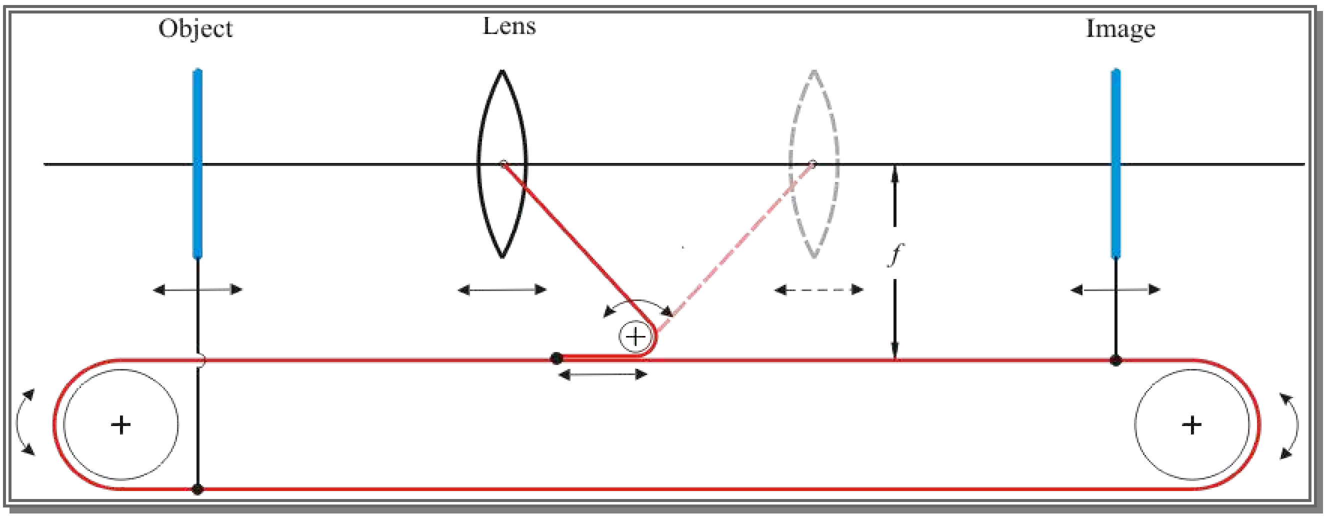


Figure 5

The belt connected to the lens loosens and allows a displacement of this element either to the right (with a magnification $M_r<1$) or to the left in a symmetrical position with respect to the initial location (with a magnification $M_l = 1/M_r > 1$).
Let d be the belt displacement:

$$\boxed{x' = f + d + \sqrt{(f+d)^2 - f^2}} \qquad \boxed{x = f + d - \sqrt{(f+d)^2 - f^2}}$$

The lens law is again satisfied:

$$\boxed{x \cdot x' = f^2}$$

The sliding rods mechanism

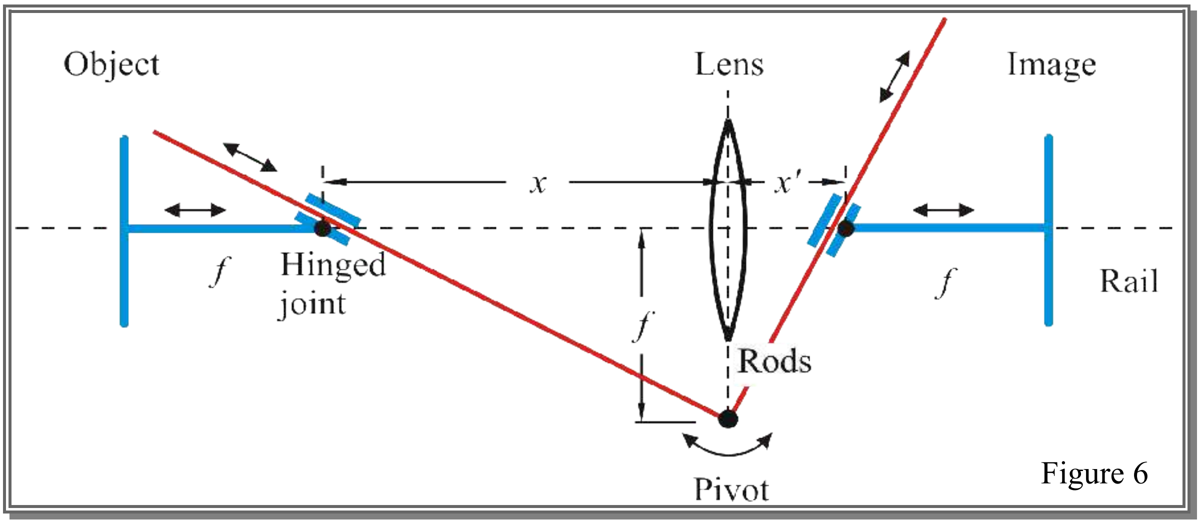


Figure 6

The device is made up of two rods joined at right angle and hinged at the vertex in a pivot located aside the lens position and distant f (the lens focal length) from the optical axis.
The rods slid into two hinged joints which are connected to the object and image planes respectively.
When the rods are turned around the pivot, the two planes move along the optic axis at which they are bounded, sliding along a rail as in the previous case. From Fig. 6:

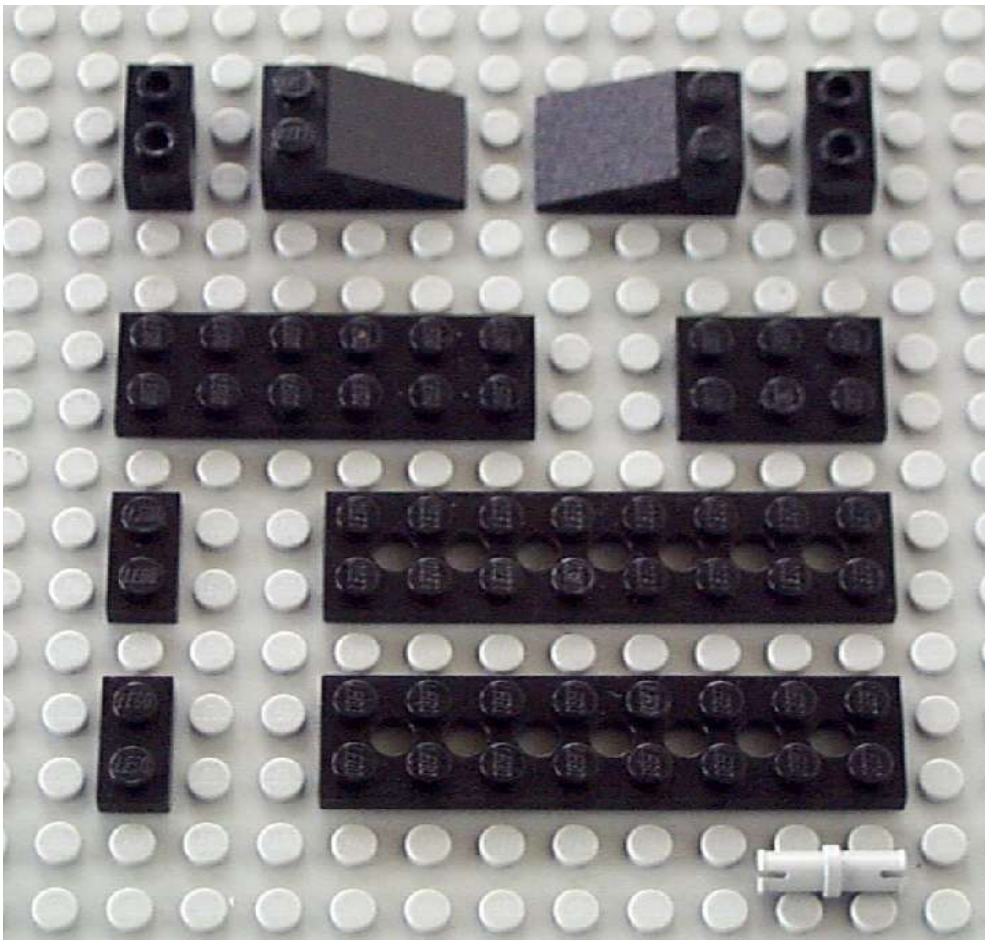
$$\boxed{\frac{x}{f} = \frac{f}{x'}}$$

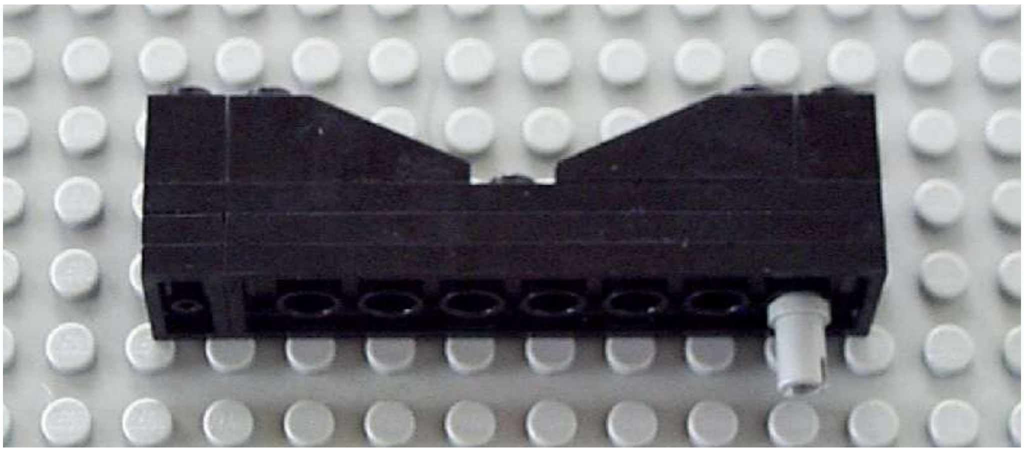
Which is again equivalent to the lens law.

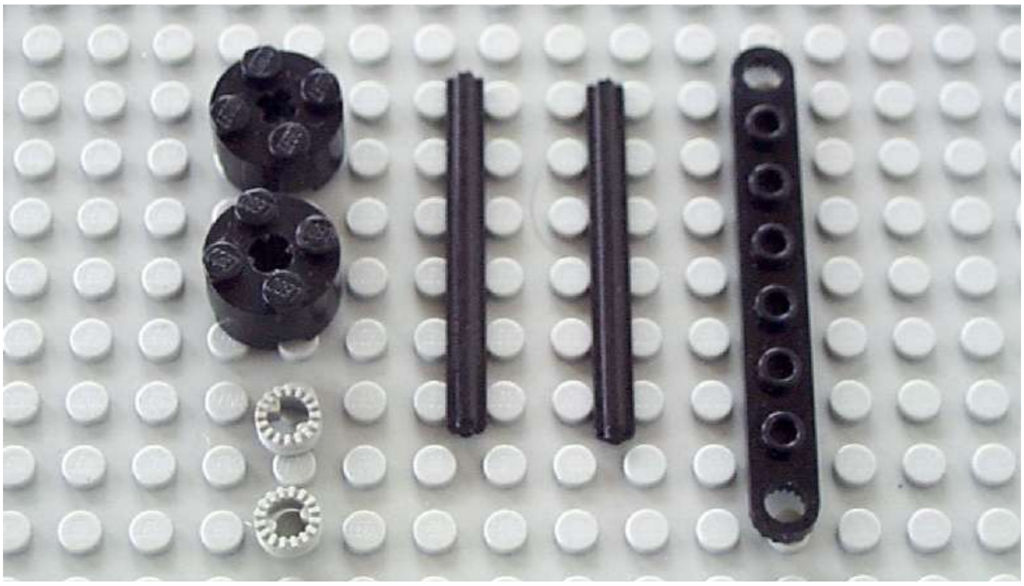
THE LENS LAW

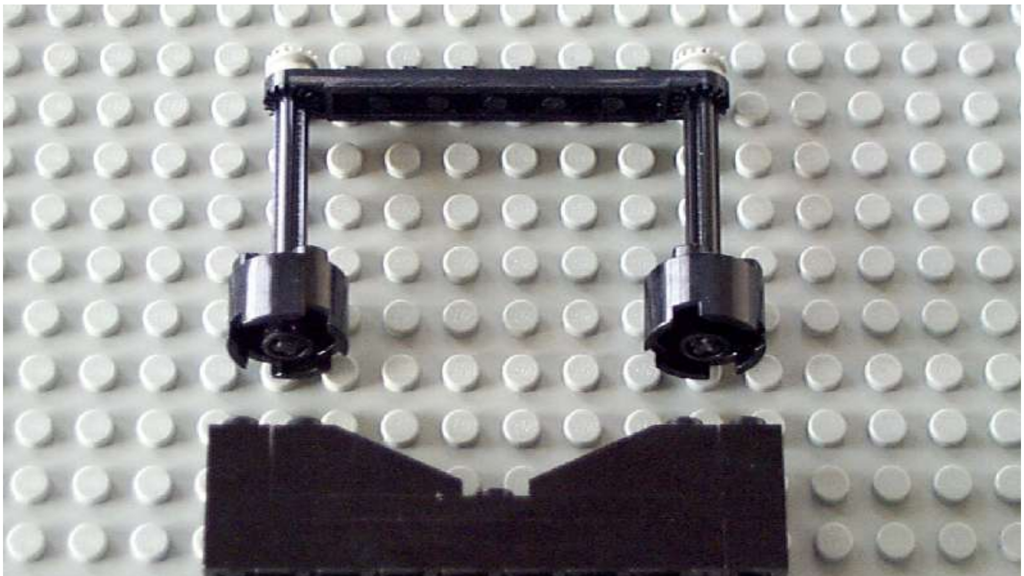
The three hinged rods autofocus mechanism

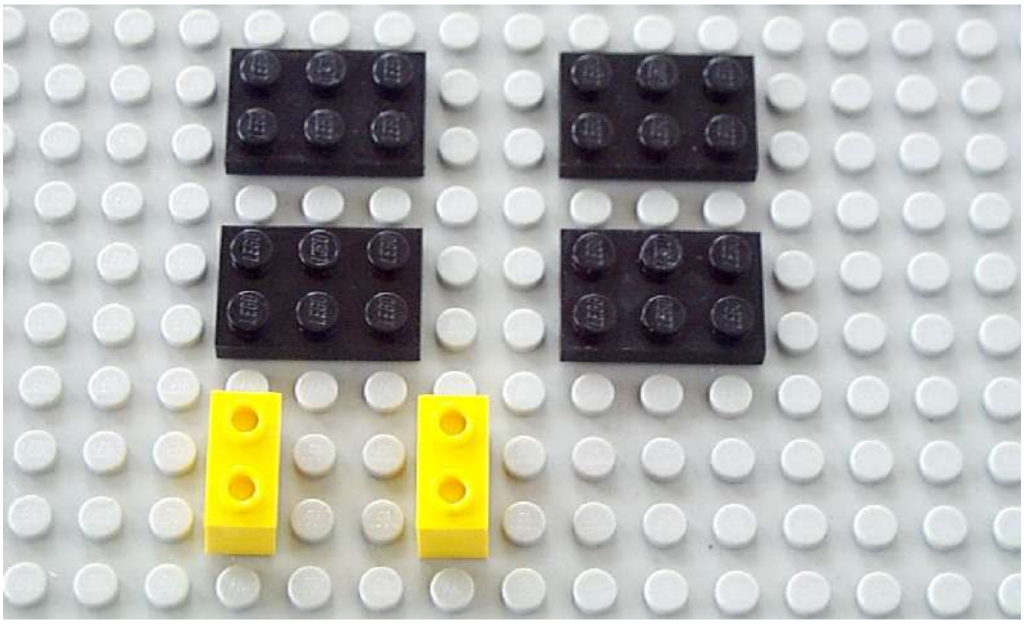
THE LENS HOLDER

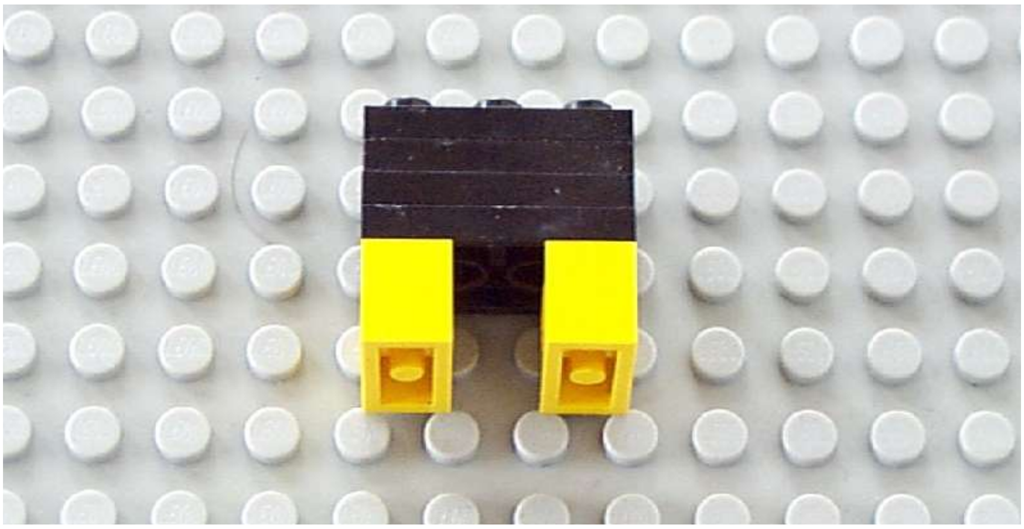


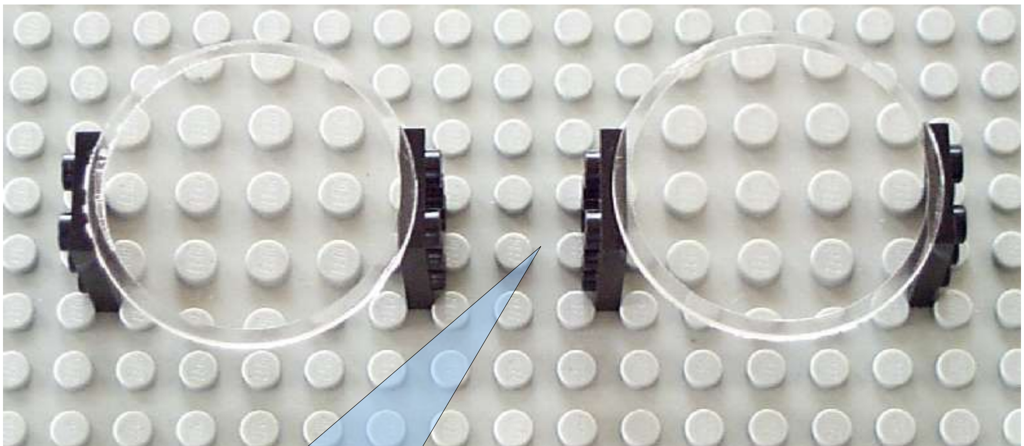






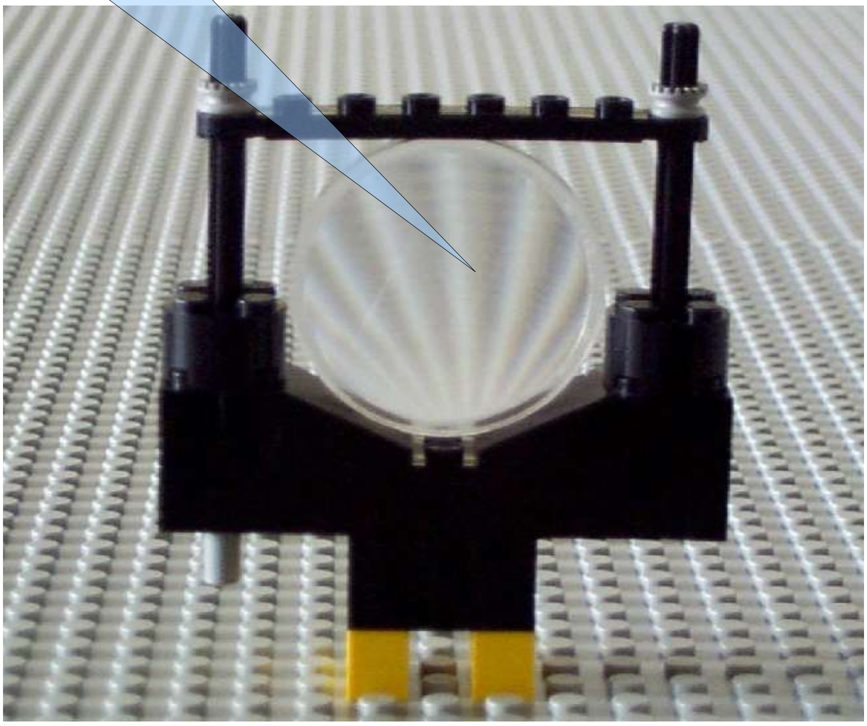




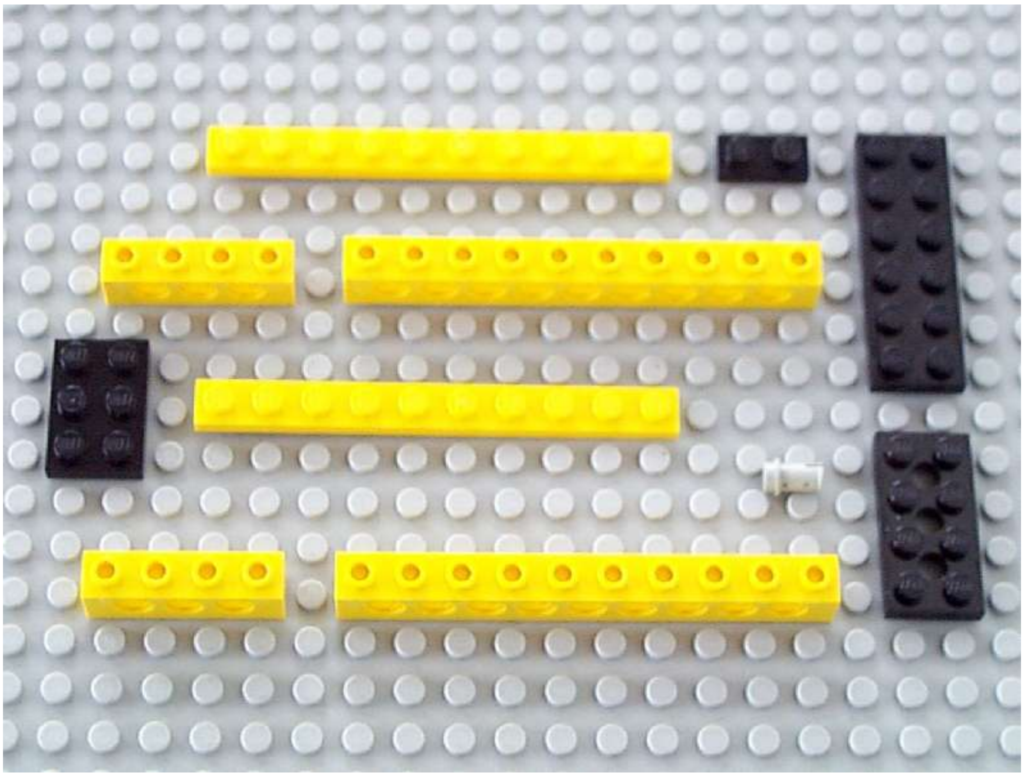


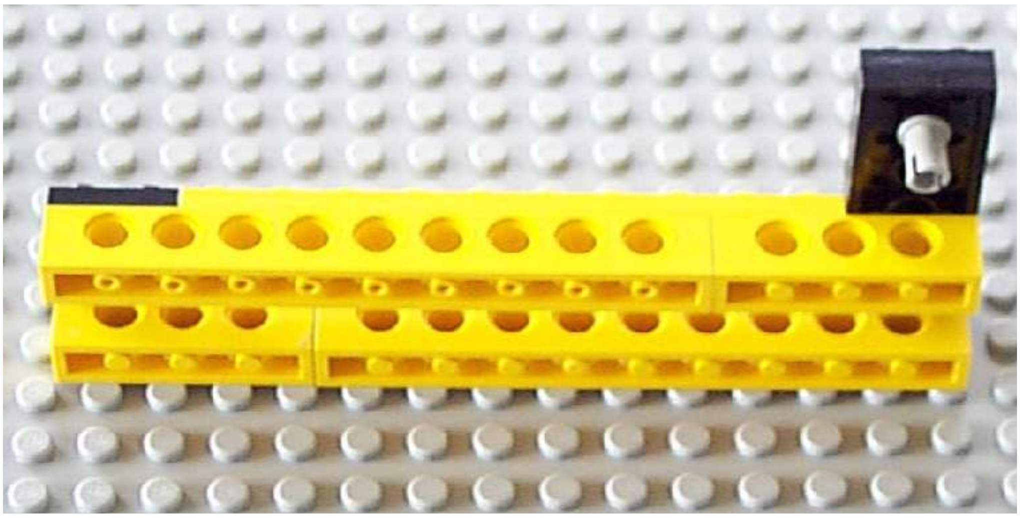
CUSTOM PIECE: TWO PLANO CONVEX PLASTIC LENSES WITH A FOCAL LENGTH OF 180 mm AND A DIAMETER OF 40 mm

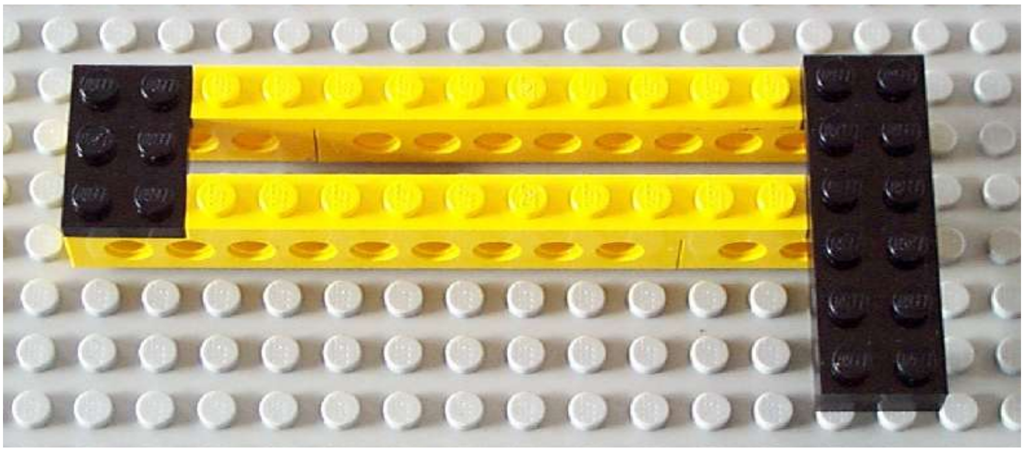
THE TWO LENSES ARE FACED
ALONG THE PLANE SURFACES
TO GET A FOCAL LENGTH OF 90mm

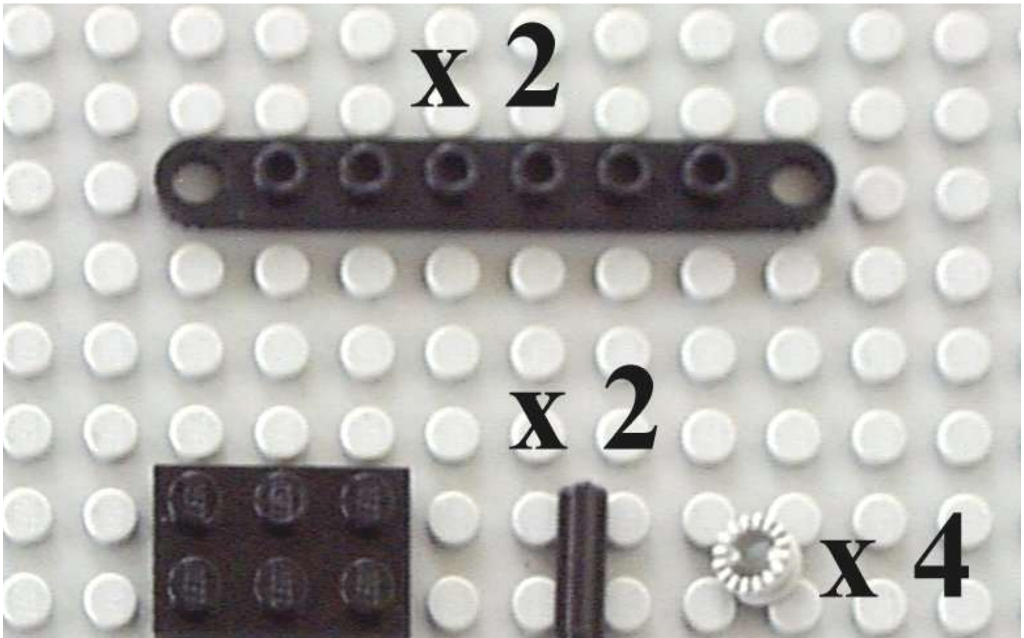


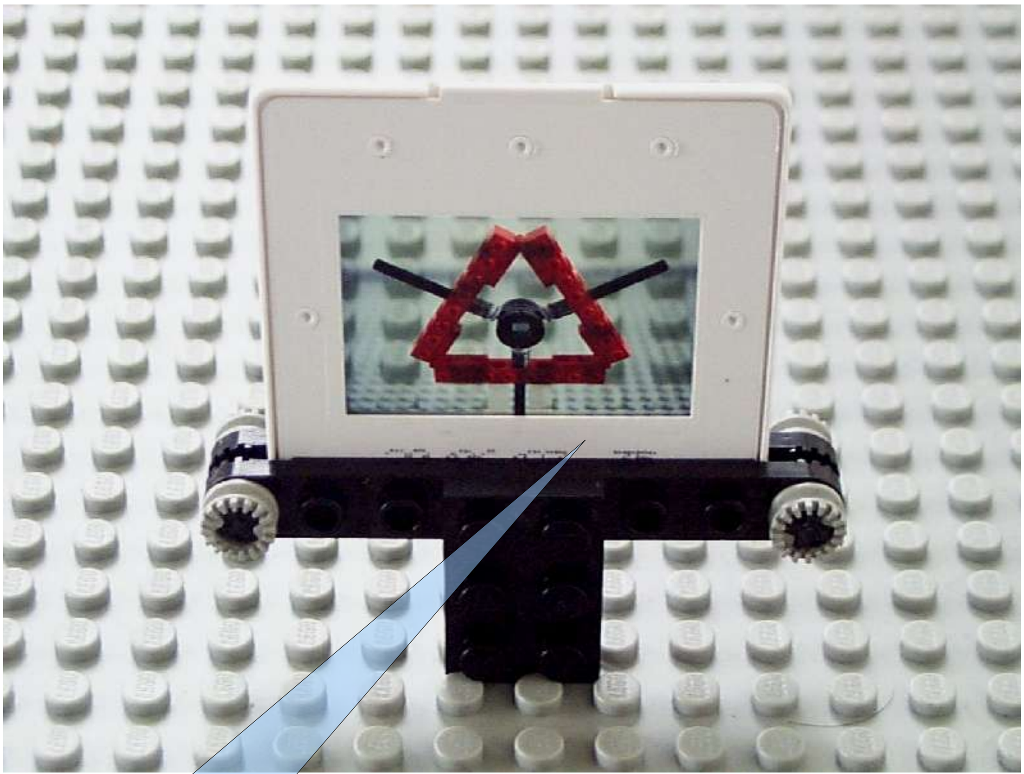
THE OBJECT STAGE



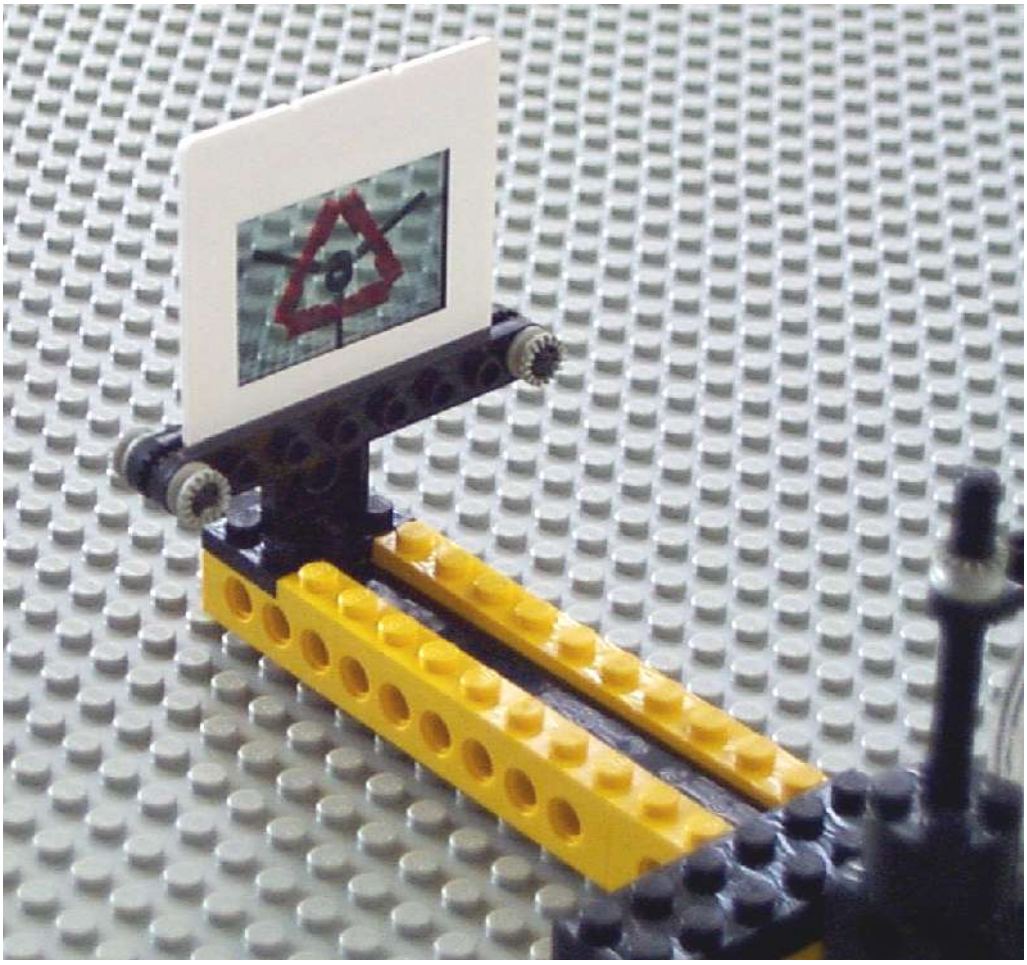




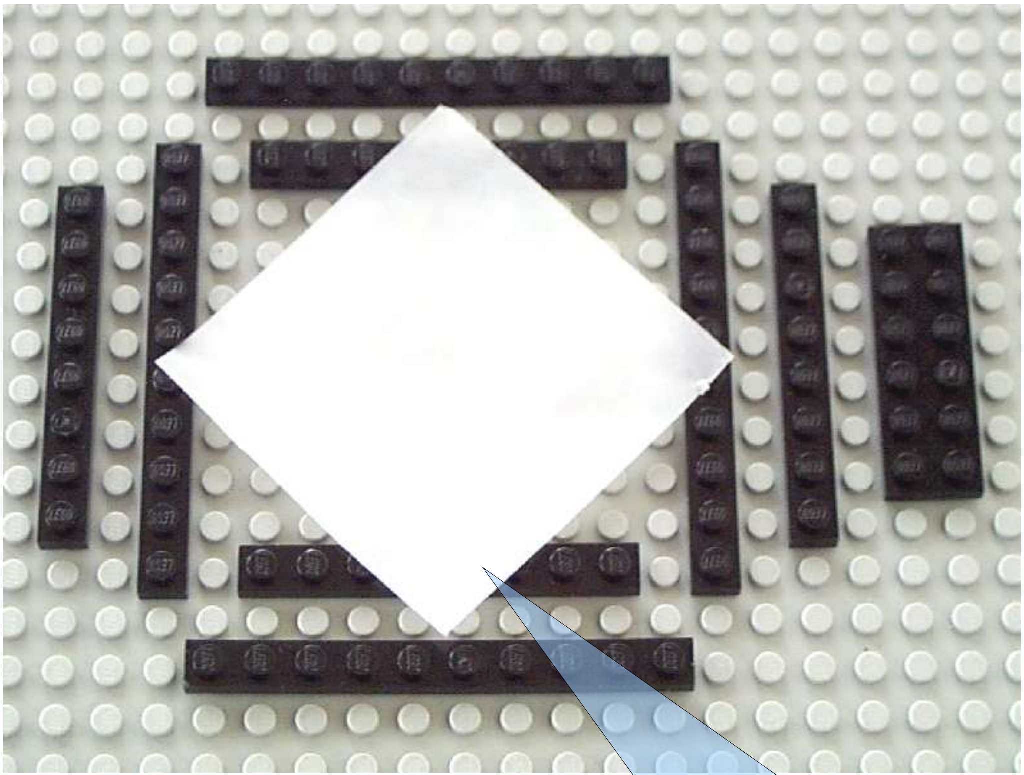

x 2
x 2
x 4

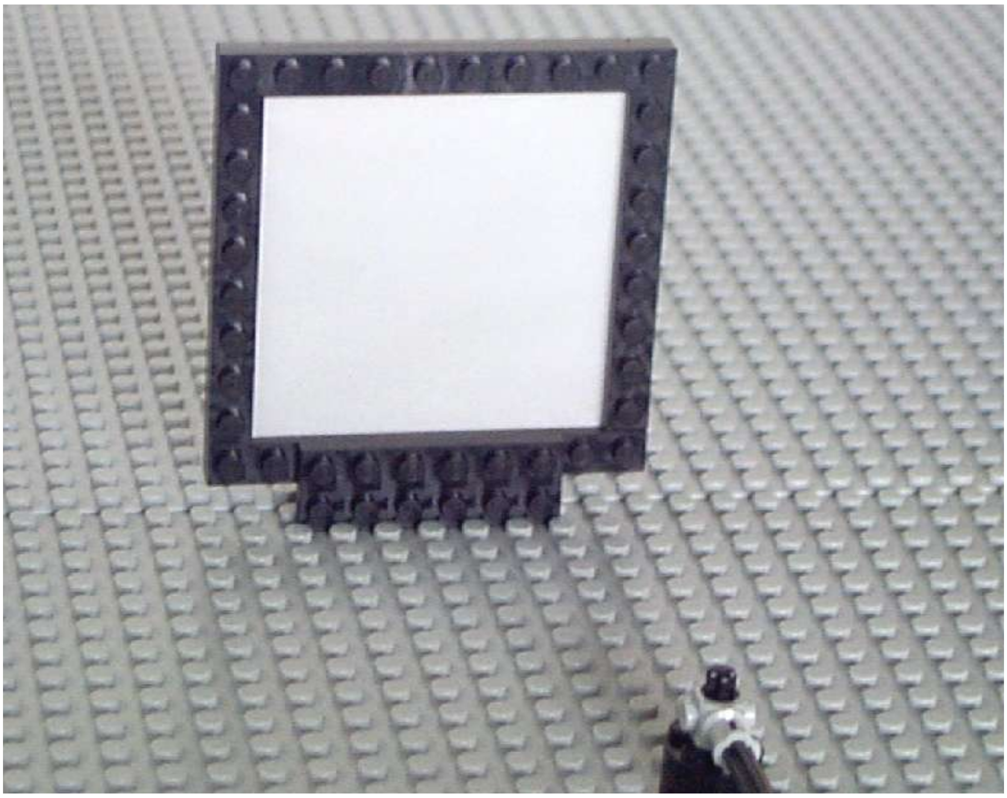


CUSTOM PIECE: AN OBJECT SLIDE

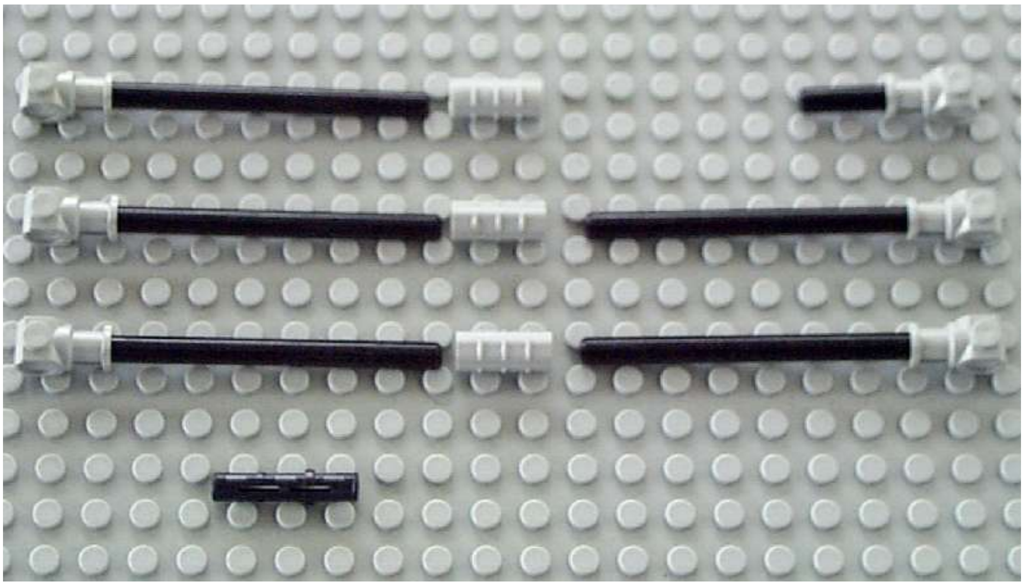


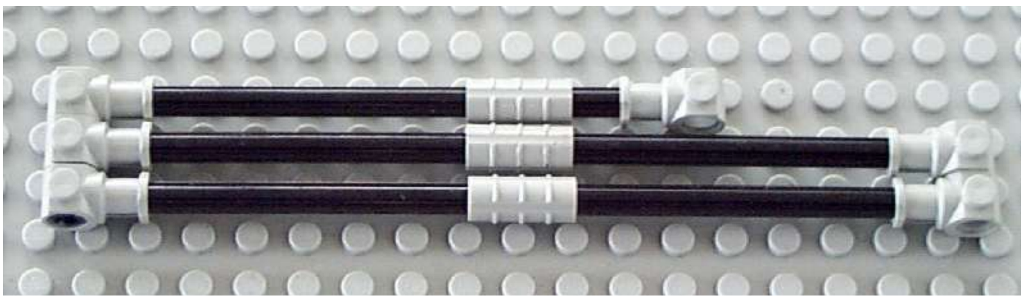
THE IMAGE PLANE

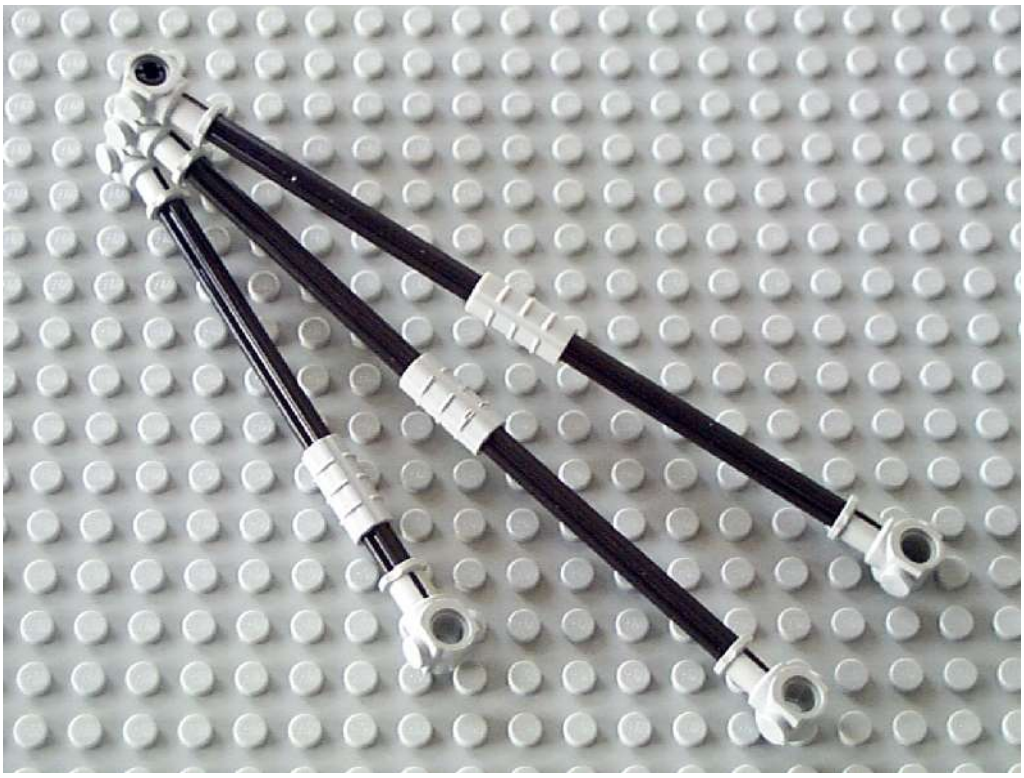




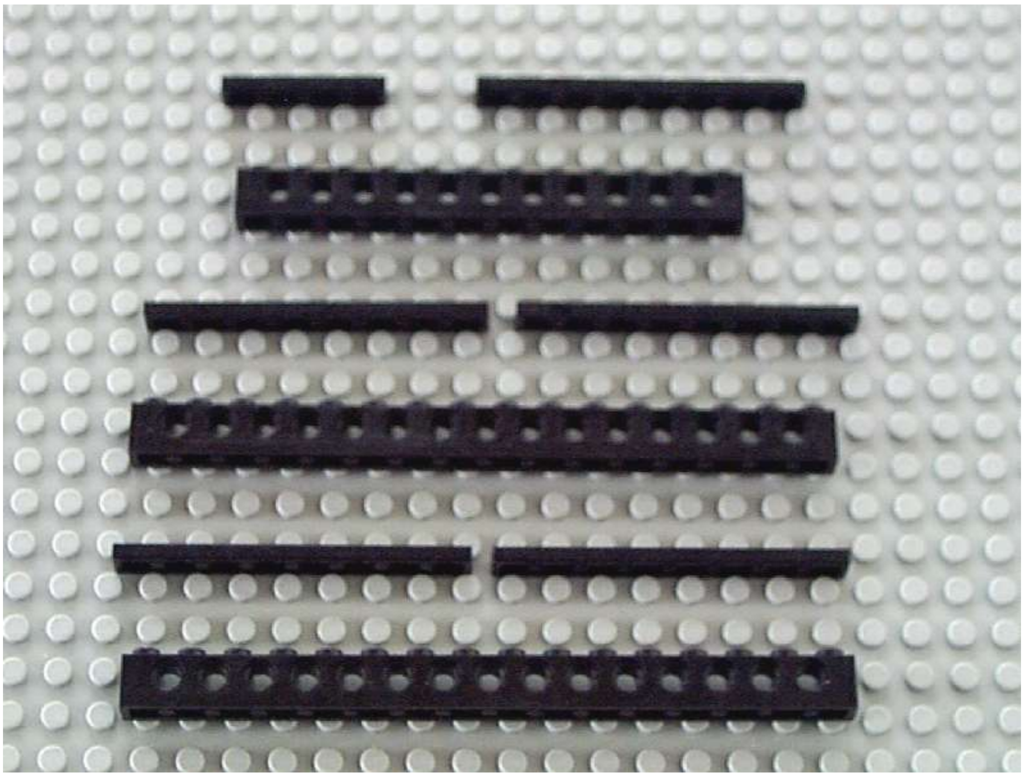
THE HINGED RODS

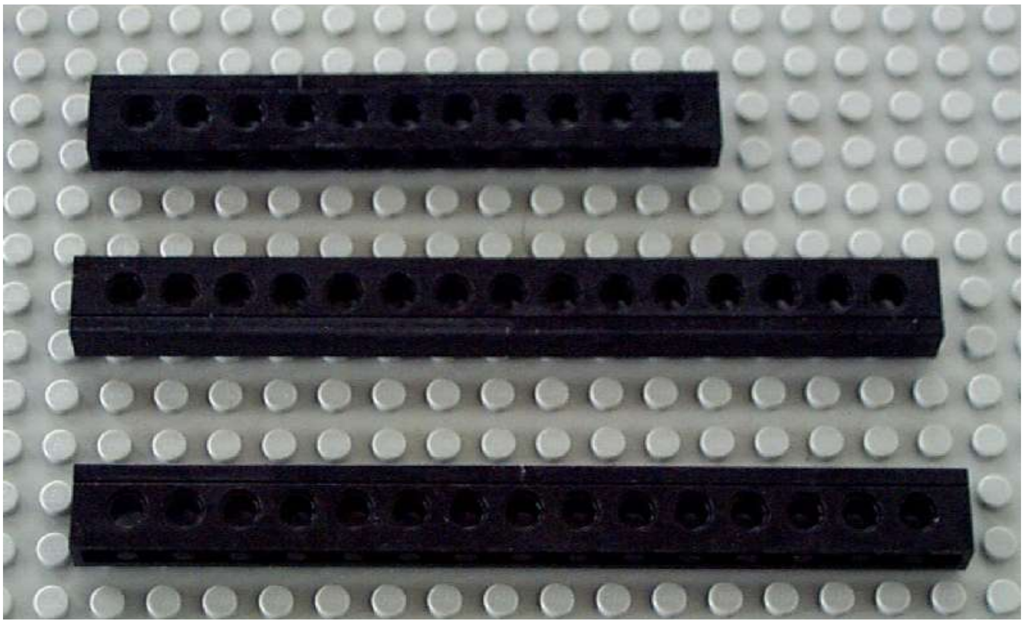




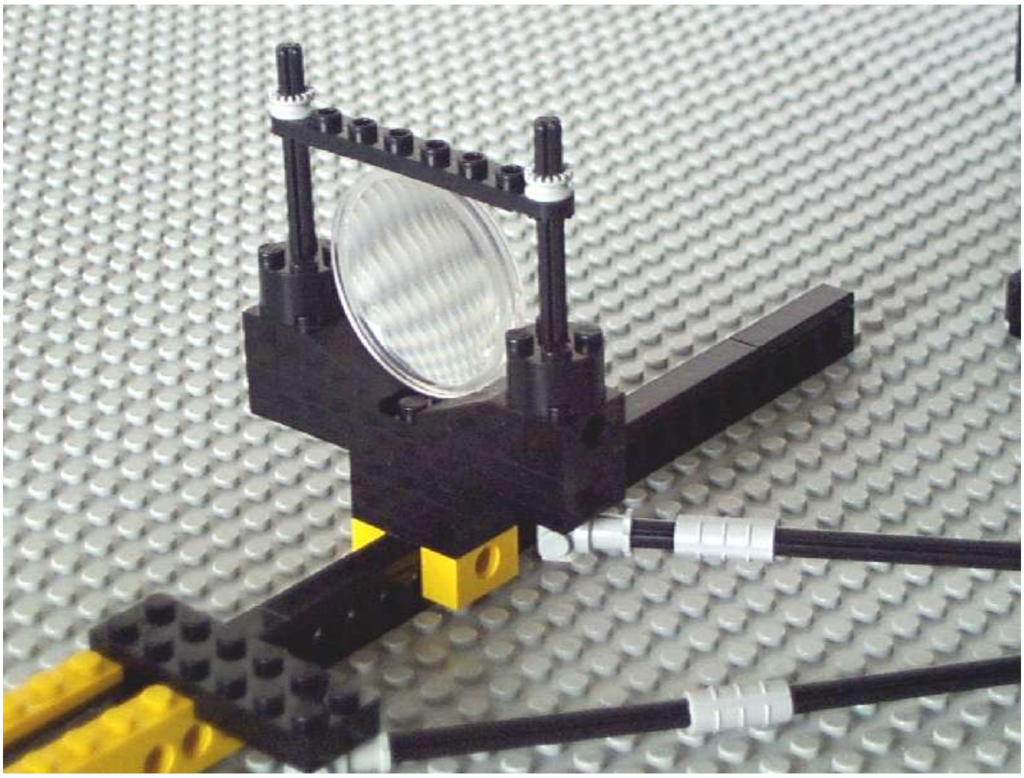


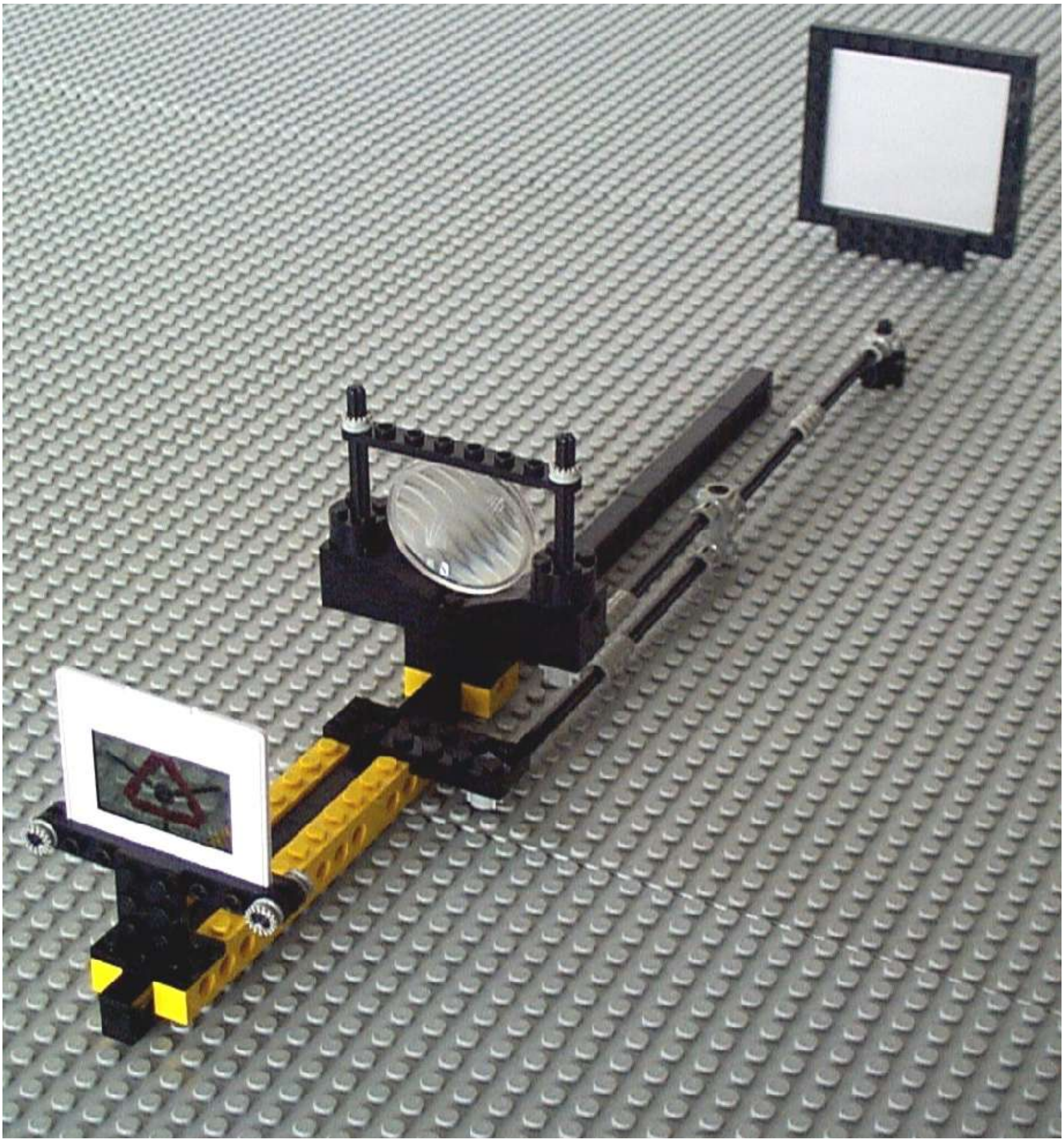
THE SLIDING RAIL





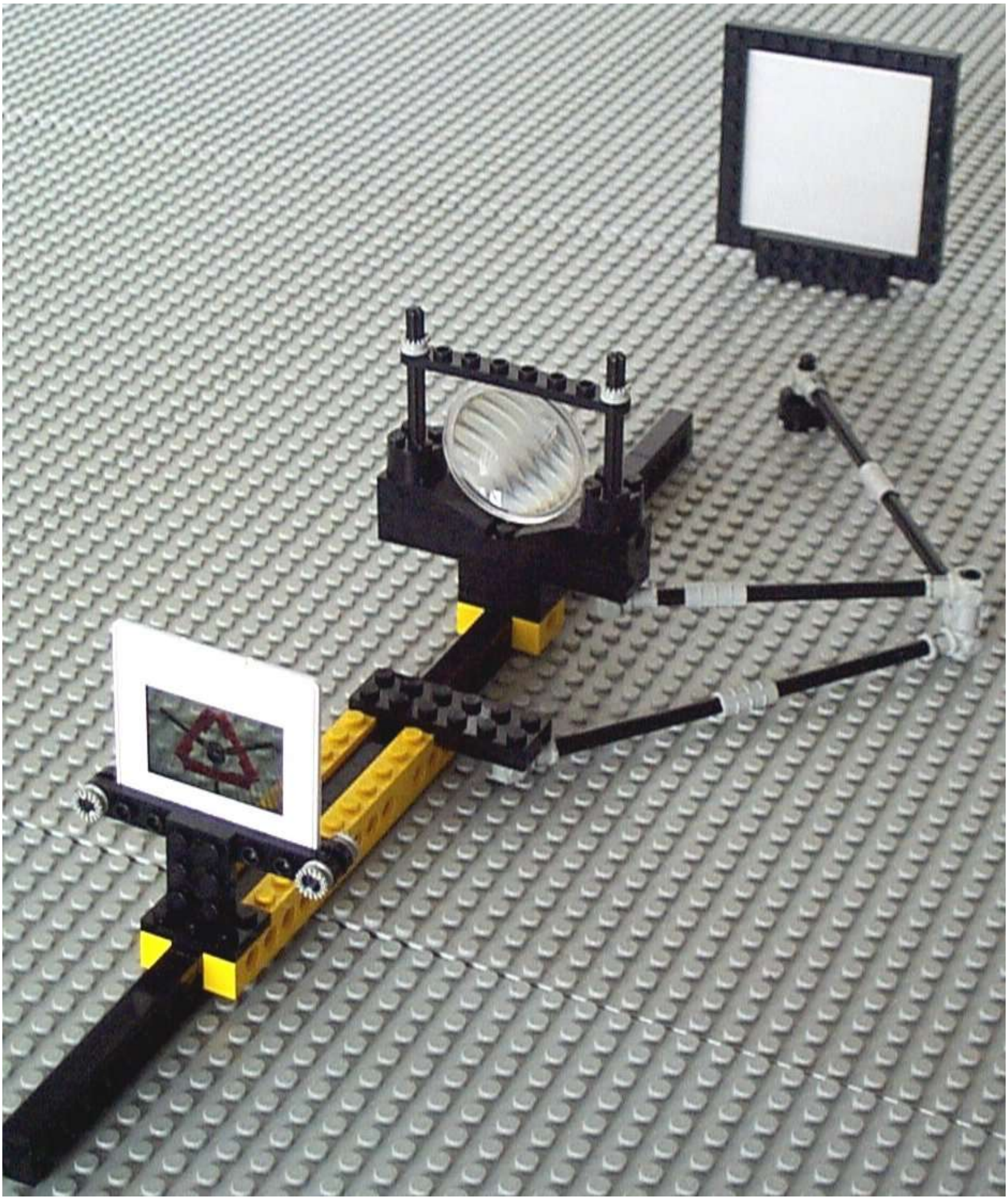
THE SYSTEM



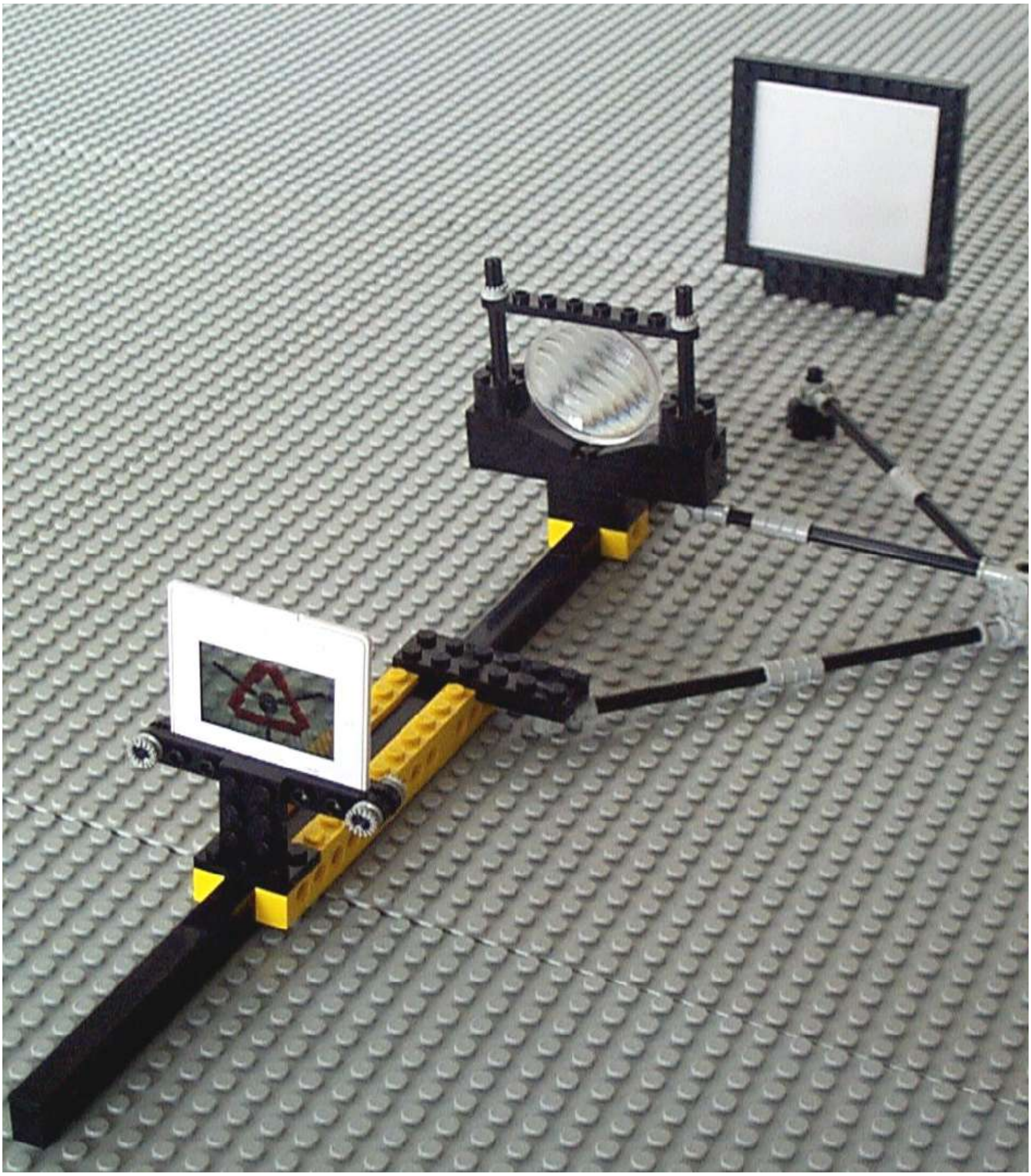


$$M_{\max} = -\frac{a+b}{f} = -\frac{f}{a-b}$$

(as in fig. 6b)

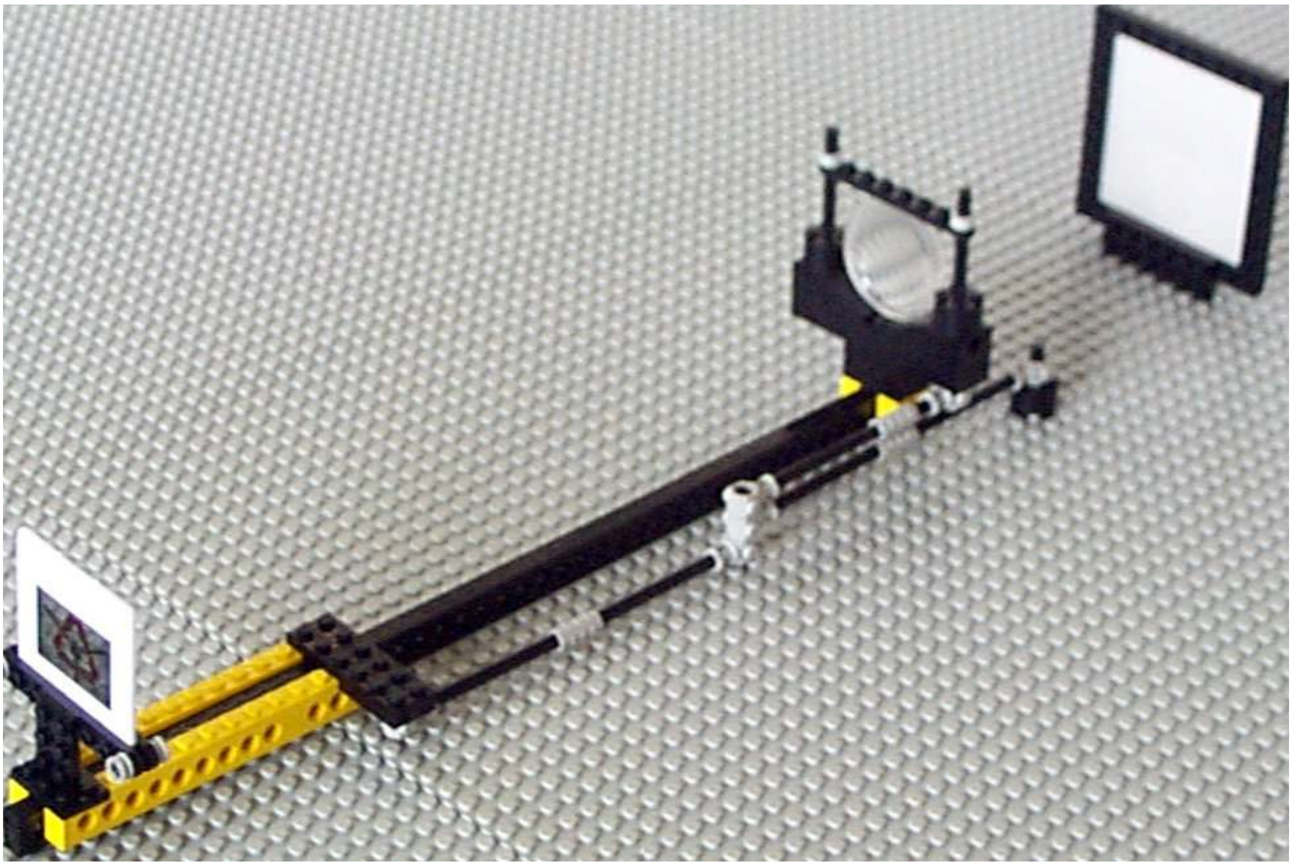


$$\boxed{M = -1}$$



$$\boxed{M = -1}$$

(as in fig. 6c)



$$\boxed{M_{\min} = -\frac{a-b}{f} = -\frac{f}{a+b}}$$

(as in fig. 6a)

COMMENTS AND CONCLUSIONS

An improvement of this set up was realized (but unfortunately there is no documentation about that) with the use of the mindstorm technology. A motor moved the optical elements periodically changing the magnification from M_{min} to M_{max} and back again. Two touch sensors were used at the two travel range ends to reverse the movement direction. A light sensor was used to detect the midrange position to pause the movement for some time at magnification M=1. A lamp and a collimating lens were used to illuminate the object slide from behind. All the process was controlled by the RCX brick.

Three autofocus techniques are reported in the introduction: *the three hinged rods*, *the endless belt*, and *the sliding rods* mechanisms. Though only *the three hinged rods* mechanism has been realized with LEGO, the other two mechanisms could be easily realized as well.

It will be interesting to stress that *the three hinged rods mechanism* derives from the Peaucellier linkage (or P. cell or P. inversor) (http://en.wikipedia.org/wiki/Peaucellier-Lipkin_linkage or, a LEGO version: http://staff.science.uva.nl/~leo/lego/peaucellier.html)

In conclusions, this experience could be a valuable kit to teach the lens law and even to introduce the concept of *analog computers,* showing how some mechanical device can perform some sort of *computing* to *solve* equations (this time the lens law equation) (see for example http://en.wikipedia.org/wiki/Analog_computer).